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Optimization of structural parameters for spatial flexible redundant manipulators with maximum ratio of load to mass

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Abstract: Optimization of structural parameters aimed at improving the load carrying capacity of spatial flexible redundant manipulators is presented in this paper. In order to increase the ratio of load to mass of robots, the cross-sectional parameters and constructional parameters are optimized respectively. The cross-sectional and configurational parameters are optimized simultaneously. The numerical simulation of a 4R spatial manipulator is performed. The results show that the load capacity of robots has been greatly improved through the optimization strategies proposed in this paper.
Key words: flexible manipulator; structural parameter; ratio of load to mass; optimal design

1　Introduction

In recent years, there has been a widespread interest in the light-weight mechanisms and manipulators with flexible members because advantages over rigid body such as: smaller actuators, lower power requirements, higher operation speed, high load to mass ratio and more compact link design etc. These have been demonstrated by a number of papers published in many areas concerning elastic mechanisms and flexible manipulators. Much advancement has been made[1-3].

The most advancement in flexible mechanisms and manipulators concentrated on dynamic modeling. It is necessary to pay more attention to the dynamic characteristics and structure design of flexible mechanisms and manipulators. Encouraging achievements have been obtained in the KED design of mechanisms by way of nonlinear optimization[6-8], optimality criteria techniques[9-11] and kinematic refinement[12]. The relationship between the natural frequency and cross-sectional parameters of the elastic mechanisms was derived theoretically and illustrated by means of numerical example[13]. However, little attention has been paid to the study of flexible manipulators. By carefully analyzing the dynamic equations of flexible manipulators, we find that the structural parameters (such as the cross-sectional parameters, the configurational parameters of links and the lumped mass at the joints) play a very important role in the dynamics of flexible manipulators. It is obvious that the dynamic performance of flexible manipulators can be improved by optimizing these design parameters.

Meanwhile, the operational tasks of today's robot manipulators have become more and more sophisticated, which requires that manipulators possess more and more degrees of freedom to of-

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fer greater dexterity and versatility. Kinematic redundancy occurs when the degrees of freedom of a manipulator are greater than the minimum number required to execute a given task. The kinematic and dynamic performance of manipulators can be greatly enhanced due to the kinematic redundancy. So, an ever increasing number of researchers have been directing their efforts towards the adoption of redundancy, resulting in many improvements. However, flexibility of links and joints has rarely been considered in most of the studies[4-5], the assumption of rigid body can no longer be applied to the new generation of robot manipulators, especially to those used in space.

The joint motion planning of flexible redundant manipulators has been proved to be effective in decreasing elastic deformation[14-15]. However, these studies are performed under the supposition that the configurational parameters and cross-sectional parameters are constant. In fact, corresponding to the prescribed trajectory of end-point and the fixed initial joint configuration, the resolution of the configurational parameters for a redundant manipulator is infinite, and the cross-sectional parameters are also changeable. The configurational parameters can be optimized to improve the dynamic performance such as, decreasing the elastic deformation error of the end-point, enhancing the ratio of load to mass etc. The cross-sectional parameters are similar.

In this paper, optimization of structural parameters to improve the load carrying capacity of spatial redundant flexible manipulators is studied. The cross-sectional parameters and configurational parameters are optimized individually, and the cross-sectional parameters and the configurational parameters are optimized simultaneously with the end-effector tracing the prescribed trajectory. The ratio of load to mass is greatly enhanced, which is demonstrated by the numerical simulation of a 4R spatial manipulator.

This paper is organized as follows. In the following sections, a theoretical analysis of the dynamics, kinematics and load capacity of flexible redundant manipulators is presented. Then, three strategies of structural parameter optimization are proposed. Finally, a numerical simulation is performed and several valuable conclusions are drawn.

2 Theoretical analysis

2.1 Dynamics

It is well known that the fatal disadvantage of flexible manipulators is the deterioration of the end-effector accuracy due to the deformation of manipulators. The end-effector motion of flexible manipulators is not only determined by the joint motion, but also influenced by the elastic deformation of links. There are basically two methods for modeling flexible manipulators, i.e. assumed mode method and finite element method (FEM)[1]. Through comparison the FEM has been showed to be more convenient and effective. Therefore, a special element, called a spatially translating and rotating beam element, was developed[17] based on FEM and is used here to drive the dynamic equations of a spatial flexible manipulator as follows

$$[\boldsymbol{M}_s]\{\ddot{\boldsymbol{U}}_s\}+[\boldsymbol{C}_s]\{\dot{\boldsymbol{U}}_s\}+[\boldsymbol{K}_s]\{\boldsymbol{U}_s\}=\{\boldsymbol{F}_s\} \ , \tag{1}$$

where $[\boldsymbol{M}_s]$,$[\boldsymbol{C}_s]$ and $[\boldsymbol{K}_s]$is the $nu\times nu$ global mass , damping and stiffness matrix, respectively. $\{\boldsymbol{F}_s\}$ is the $nu\times 1$ generalized force vector. $[\boldsymbol{U}_s]$,$[\dot{\boldsymbol{U}}_s]$ and $\{\ddot{\boldsymbol{U}}_s\}$ are the generalized displacement, velocity and acceleration vector of the elastic motion of flexible links, respectively. nu is the number of the coordinates.

In the dynamic equations of spatial flexible manipulators, we can find that the terms of $[\boldsymbol{M}_s]$,$[\boldsymbol{C}_s]$,$[\boldsymbol{K}_s]$,$[\boldsymbol{F}_s]$ are determined by not only the joint motion of robot, but also the structural parameters such as the configurational parameters (link length) and cross-sectional parameters (the height and width of links). Therefore, two strategies can be taken to improve the dynamic performance of flexible manipulators when keeping the end-effector tracing the prescribed trajectory. One is planning the joint motion q, $\dot{q}$ and $\ddot{q}$ by the use of kinematic redundancy as was done in[14-15]. The other is optimizing structural parameters of robots. The latter strategy will be investigated in this paper.

2.2 Kinematics

The relation between the task space variables $x \in R^m$ and joint space variables $q \in R^m$ of redundant manipulators can be defined as

$$\boldsymbol{x} = \boldsymbol{\varphi}(\boldsymbol{q}) \ , \tag{2}$$

where $\boldsymbol{q}$ is the $n \times 1$ vector of joint variables. $\boldsymbol{x}$ is the $m \times 1$ vector of task variables. $\boldsymbol{\varphi}$ is a differentiable nonlinear vector function whose structure and parameters are assumed to be known for any given manipulator.

For a redundant manipulator, the number of joint space coordinates exceeds the number of workspace coordinates m. The effect of the extra degrees of freedom can be easily seen in the differential relation of equation (2) that given by the manipulator Jacobian

$$\dot{x} = \boldsymbol{J}(q)\dot{q} \ , \tag{3}$$

Where, $\boldsymbol{J}$ is the $m \times n$ configuration dependent Jacobian matrix with $m < n$ formally defined as $\partial\boldsymbol{\varphi}/\partial q$. The upper dot here denotes time derivatives.

Differentiating (3), we obtain the acceleration equation

$$\ddot{x} = \boldsymbol{J}(\boldsymbol{q})\ddot{q} + \dot{J}(\boldsymbol{q})\dot{q} \ , \tag{4}$$

It is clear that in the case of a redundant manipulator with respect to a given task ($m < n$), the inverse kinematic problem has infinite solutions. From equation (4), a general formula of the inverse kinematics at acceleration level as the sum of a particular and homogeneous component can be presented as

$$\ddot{q} = \boldsymbol{J}^{+}(\ddot{x} - \dot{\boldsymbol{J}}\dot{q}) + (I - \boldsymbol{J}^{+}\boldsymbol{J})\boldsymbol{\varepsilon} \ , \tag{5}$$

Where $\boldsymbol{J}^{+} \in \boldsymbol{R}^{n \times m}$ is the pseudoinvers of the Jacobian matrix $\boldsymbol{J}$. $\boldsymbol{I} \in \boldsymbol{R}^{n \times m}$ is a unit matrix. $\boldsymbol{\varepsilon}$ is the arbitrary vector of null space of the redundant robot. $(I - \boldsymbol{J}^{+}\boldsymbol{J})\boldsymbol{\varepsilon} \in N(\boldsymbol{J})$ is the homogeneous solution which is orthogonal with $\boldsymbol{J}^{+}(\ddot{x} - \dot{J}\dot{q}$, The homogeneous solution indicates the self-motions among links. This means that there are, in general, an infinite number of ways to configure the arms for a given position/orientation of the end-effector. The links can move freely while maintaining the end-effector in one position/orientation. This "self-motion" capability is an attractive feature of redundant manipulators. It is the basis for the motion planning strategies to improve the dynamic performance of manipulators.

For a flexible manipulator, the end-effector position/orientation is not only the function of joint motion angle vectors, but also the function of joint and link deformations[14-15]. In this case, equations (2)~(4) become

$$x = \varphi(\boldsymbol{q}, u) \ , \tag{6}$$

$$\dot{x} = \boldsymbol{J}\dot{q} + \boldsymbol{J}_u\dot{u} \ , \tag{7}$$

$$\ddot{x} = \boldsymbol{J}\ddot{q} + \dot{J}\dot{q} + J_u\ddot{u} + \dot{J}_u\dot{u} \ . \tag{8}$$

Similar to the rigid one, a general solution of the inverse kinematics of flexible redundant manipulators at the acceleration level can be written as the sum of the particular and homogeneous components

$$\ddot{q} = \boldsymbol{J}^{+}(\ddot{x} - \dot{J}\dot{q} - J_u\ddot{u} - \dot{J}_u\dot{u}) + (I - \boldsymbol{J}^{+}\boldsymbol{J})\boldsymbol{\varepsilon} \ , \tag{9}$$

Where $u \in R^{nl}$ is the generalized coordinate, which describes the deformation of joints and links. nl is the number of the flexible degree of freedom. $\boldsymbol{J}_u = \partial \boldsymbol{\varphi} / \partial u \in R^{n \times nl}$ is the flexible Jacobian matrix. $(-\boldsymbol{J}_u \ddot{u} - \dot{J} \dot{u})$ is the elastic deformation compensation term which is used to compensate the end-effector deformation errors. In this way, the end-effector can follow the track of the trajectory precisely. However, the compensation term brings out an unexpected sharp angular acceleration fluctuation, which is difficult to control in practice. The main aim of this study is to investigate the optimization of structural parameters of redundant flexible manipulators. Therefore, this term can be neglected here.

2.3 Load capacity

When the end-effector motion of a manipulator is satisfied, the ratio of maximum load M_p that manipulators can carry to its mass M_r is called as the ratio of load to mass μ. It is an important index to evaluate the performance of flexible manipulators.

The mass of manipulators can be written as follows

$$M_r = \sum_{i=1}^{n} (\rho_i b_i h_i l_i + m_{ai}) \quad , \tag{10}$$

where, ρ_i is the mass density of the i-th link. l_i, b_i and h_i are the length, cross-sectional width and height of the i-th link, respectively. m_{ai} is the lumped mass at the i-th joint.

Therefore, the ratio of load to mass μ can be expressed as

$$\mu = M_p / M_r = M_p / \sum_{i=1}^{n} (\rho_i b_i h_i l_i + m_{ai}) \quad , \tag{11}$$

Obviously, there are two strategies that can be taken to improve the load carrying capacity of flexible manipulators. One is the strategy of joint motion planning by which the joint motion is optimized to enhance the maximum load that the manipulators can carry. This will be discussed in another paper. The other is the strategy of parameter optimization by which the structural parameters (such as the cross-sectional parameters b_i and h_i, the configurational parameters l_i, the lumped mass m_{ai}) are optimized to decrease the mass of a manipulator. This strategy will be studied comprehensively in this paper.

3 Optimization strategies

There are two kinds of parameters in the design of flexible manipulators. One is a cross-sectional parameter, the other is a configurational parameter. They are discussed as follows.

3.1 Optimization of cross-sectional parameters

It can be seen from the dynamic equation (1) of flexible manipulators that the terms of $[\boldsymbol{M}_s]$, $[\boldsymbol{K}_s]$ are the function of cross-sectional parameters, i. e. the width and height of link cross-sections, if the joint motion and link length of manipulators are fixed. From the modeling process of dynamic equations, we can understand that the variation of the cross-sectional parameter will change the system mass matrix, stiffness matrix and generalized force vector. This may leads to a change in the dynamic performance of manipulators. Therefore, by optimizing the cross-sectional parameters, the manipulator system mass can be distributed reasonably among links, and as a result, the stiffness of the manipulator system can be enhanced and the gross mass of manipulator can be reduced.

In the process of optimization, it is thought that the lumped mass m_{ai} at joints will be decreased with the decrease in the link and actuator masses, assumed to be determined by

$$m_{ai} = m_{ai0} + \varphi_i(\tau_i) \quad , \tag{12}$$

where m_{ai0} is the basic lumped mass at the i-th joint. τ_i is the maximum driving torque of the i-th actuator. φ_i is the function between m_{ai} and τ_i, which can be supposed to be a linear relationship.

Meanwhile, the deformation error of the end-effector and the stress of links may increase with the decrease in link mass. The constraints shown in follows should be satisfied

$$\sigma_{i\max} \leqslant [\sigma_i] \ , \tag{13}$$

$$\delta_{\max} \leqslant [\delta] \ , \tag{14}$$

where $\sigma_{i\max}$ is maximum stress of the i-th link, $\delta_{\max}$ the maximum defamation error of end-effector, $[\sigma_i]$ is the allowable stress of the i-th link, $[\delta]$ is the allowable error of end-effector.

The term of M_r/M_p can now be supposed to be the optimization objective of this study to decrease the mass of the manipulator. The cross-sectional parameters, the width b_i and height h_i of link cross-section, are selected to be optimization variables. The strategy of cross-sectional parameter optimization for flexible manipulators can be expressed in the following mathematical form

$$\begin{aligned} \text{Min} \quad & f(x) = 1/\mu(x) = M_r(x)/M_p \\ s.t. \quad & \sigma_{i\max} - [\sigma_i] \leqslant 0 \quad (i = 1, \cdots, n) \\ & \delta_{\max} - [\delta] \leqslant 0 \\ & x_l \leqslant x \leqslant x_u \ , \end{aligned} \tag{15}$$

where $x = (x_1, \cdots, x_{2n}) = (b_1, \cdots, b_n, h_1, \cdots, h_n)$ denotes the cross-sectional parameters, n is the number of links, x_l and x_u are the lower and upper border of x, respectively.

3.2 Optimization of configurational parameters

The cross-sectional parameters are optimized to improve the load carrying capacity in *section* 3.1. As a matter of fact, the configurational parameters of redundant manipulators are also changeable, and so can be adjusted through optimization.

According to the analysis in section 2, the geometric model of a redundant manipulator can be written in the following form

$$x_0 = \boldsymbol{\varphi}(q_0, l) \ , \tag{16}$$

Since equation (12) represents equations with n unknowns, there are an infinite number of solutions for $l = (l_1, l_2, \cdots, l_n)$. Different l will lead to different joint motion $\boldsymbol{q}, \dot{q}$ and $\ddot{q}$ if x_0 and q_0 are fixed. As a result, the term of $[M_s]$, $[\boldsymbol{C}_s]$, $[\boldsymbol{K}_s]$ and $[\boldsymbol{F}_s]$ in dynamic equation (1) will vary, and the dynamic performance of manipulators may be affected. At the same time the matrices of $[\boldsymbol{M}_s]$, $[\boldsymbol{C}_s]$ and $[\boldsymbol{K}_s]$ can be directly varied with the change of the constructional parameters. So, the configuration parameters can be optimized to decrease the gross mass and improve the load carrying capacity of redundant flexible manipulators.

The configurational parameters l are selected as optimization variables. M_r/M_p is supposed to be optimization objective. The strategy of configurational parameters optimization can be written in the following form

$$\begin{aligned} \text{Min} \quad & f(x) = 1/\mu(x) = M_r(x)/M_p \\ s.t. \quad & \sigma_{i\max} - [\sigma_i] \leqslant 0 \quad (i = 1, \cdots, n) \\ & \delta_{\max} - [\delta] \leqslant 0 \\ & x_l \leqslant x \leqslant x_u \ , \end{aligned} \tag{17}$$

where $x = (x_1, \cdots, x_n) = (l_1, \cdots, l_n)$ represents the configurational parameters of manipulators.

3.3 Comprehensive optimization of the cross-sectional and configurational parameters

As mentioned above, either cross-sectional parameter or configurational parameter can vary the terms of $[\boldsymbol{M}_s]$, $[\boldsymbol{C}_s]$, $[\boldsymbol{K}_s]$ and $[\boldsymbol{F}_s]$ in dynamic equation (1). Each of them has important effects on the dynamic performance of flexible manipulators. In section 3.1 and section 3.2, the configurational and cross-sectional parameters are optimized individually to improve the ratio of load to mass of manipulators. In this section, a comprehensive optimization of the configurational parameters and cross-sectional parameters is presented. It may be more effective to decrease the gross mass and improve the ratio of load to mass of flexible manipulators. In this optimization strategy, both the configurational parameters and cross-sectional parameters are selected as optimization variables simultaneously. The maximum ratio of load to mass is supposed to be optimization objective. Therefore, this comprehensive optimization strategy can be written in the following mathematical form

$$\begin{aligned} \text{Min} \quad & f(x)=1/\mu(x)=M_r(x)/M_p \\ s.t. \quad & \sigma_{i\max}-[\sigma_i]\leqslant 0 \quad (i=1,\cdots,n) \\ & \delta_{\max}-[\delta]\leqslant 0 \\ & x_l\leqslant x\leqslant x_u \end{aligned} \quad , \tag{18}$$

where $x(x_1,\cdots,x_{3n})=(l_1,\cdots,l_n,b_1,\cdots,b_n,h_1,\cdots,h_n)$.

4 Simulation and discussion

A spatial 4R manipulators with four flexible links (shown in Fig. 1) is used in the numerical solution for the design of robot based on the three strategies proposed above. The spatial robot has one degree of redundant freedom if only the position of end-effector is considered. The original parameters of a robot are given as follows: the length of each link is 200 mm. Both the cross-sectional height and the width of each link are 7 mm. Each link is made of steel with elastic modulus of 200 MPa, shear modulus of 80 MPa, and mass density of 7 800 kg/m^3. The lumped mass at each joint is 40 g and the lumped mass at the endpoint is supposed to be 20 g.

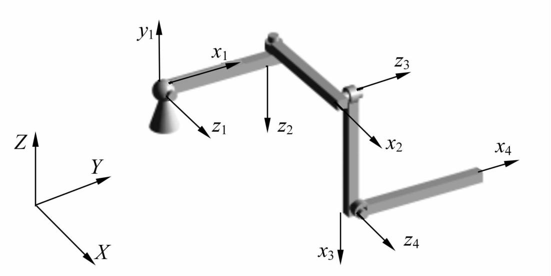

Fig. 1 A spatial 4R flexible manipulator

It is supposed that the end-effector runs along a straight line trajectory of 0. 1 m long within one second. The motion profile of the end-effector can be expressed as

$$x=a_0+a_1t+a_2t^2+a_3t^3 \quad , \tag{19}$$

$$y=b_0+b_1t+b_2t^2+b_3t^3 \quad , \tag{20}$$

$$z=c_0+c_1t+c_2t^2+c_3t^3 \quad . \tag{21}$$

where a_i, b_i, c_i $(i=0,1,2,3)$ are the coefficients determined by the boundary conditions of motion. The velocities of the end-effector at the beginning and the end of the motion are supposed to be zero.

Tab. 1 Optimization A

parameters	a_1	a_2	a_3	a_4	M_r	μ
original	7.00	7.00	7.00	7.00	375.76	0.133
optimization A	7.74	5.52	5.00	5.00	273.22	0.183

Tab. 2 Optimization B

parameters	b_1	b_2	b_3	b_4	h_1	h_2	h_3	h_4	M_r	μ
original	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	375.76	0.133
optimization B	9.18	6.32	5.00	5.00	5.00	5.23	5.00	5.00	248.76	0.201

Tab. 3 Optimization C

parameters	l_1	l_2	l_3	l_4	M_r	μ
original	200	200	200	200	375.76	0.133
optimization C	350.24	151.06	130.5	50.0	325.11	0.154

Tab. 4 Optimization D

parameters	b_1	b_2	b_3	b_4	h_1	h_2	h_3	h_4	M_r	μ
optimization C	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	325.11	0.154
optimization D	6.21	4.00	4.00	4.00	4.15	4.00	4.00	4.00	134.24	0.357

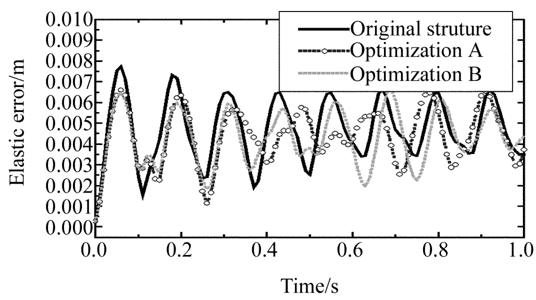

Fig. 2 Elastic error at the end-point of the manipulator

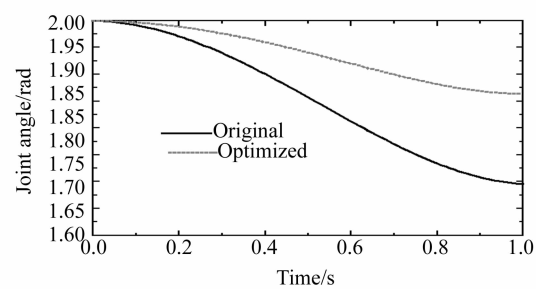

Fig. 5 Angle of 4th joint

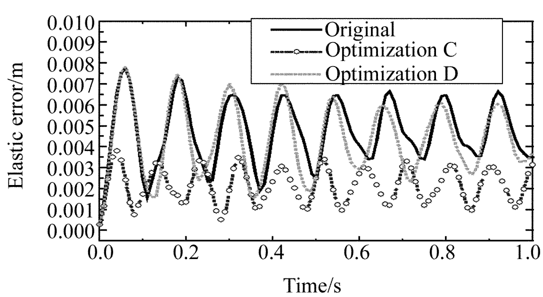

Fig. 3 Elastic error at the end-point of the manipulator

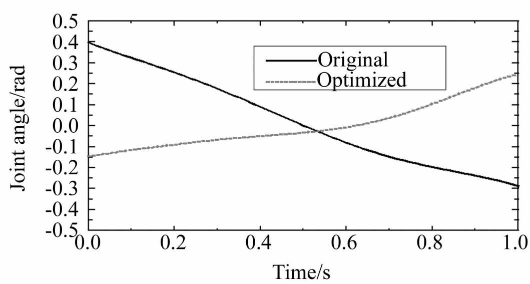

Fig. 6 Angular acceleration of the 1st joint

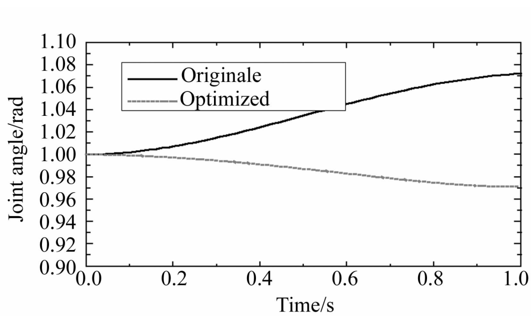

Fig. 4 Angle of the 1st joint

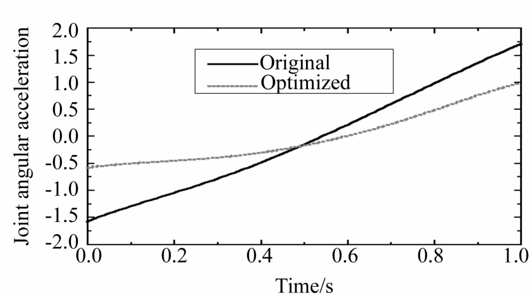

Fig. 7 Angular acceleration of 4th joint

The initial values of the joint configuration vector $\boldsymbol{q}_0$ and the arbitrary vector are set to be $(2.0, 2.0, 2.0, 2.0)^T$ and $(0.0, 0.0, 0.0, 0.0)^T$. The initial end-effector position vector $\boldsymbol{x}_0 = (a_0, b_0, c_0)^T$ can be derived from equation (2), and the end-effector trajectory is planned according equations (15)-(17). The maximum ratio of load to mass or the minimum mass of the manipulator is supposed to be optimization objective. The cross-sectional parameters and configurational parameters are optimized, individually at first, and then optimized simultaneously. The numerical results are shown in Tab. 1～Tab. 4, and Fig. 2～Fig. 7. The length unit is mm, and mass unit is g in the tables above.

In the strategy of cross-sectional parameter optimization, for comparison, two processes have been shown in Tab. 1 and Fig. 2. In the first process, i. e. optimization A, the cross-section of links is supposed to be square, and only the dimensions of cross-section, $a_i(i=1,2,3,4)$, is optimized. It can be seen that the mass of the flexible manipulator has been reduced from 375. 76 g to 273. 22 g. The ratio of load to mass has been increased by 37. 6%, from 0. 133 to 0. 183 while the end-effector error caused by elastic deformation does not increase. In the second process, i. e. optimization B, the link cross-section is selected as rectangular, both link width and height, $b_i, h_i(i=1,2,3,4)$, are optimized. We can find that the mass of the manipulator has been reduced from 375. 76 g to 248. 76 g. The ratio of load to mass increased by 51. 1% from 0. 133 to 0. 201, increased by about 10% as compared with optimization A. As shown in Tab. 1～Tab. 2, the cross-section parameters of links are quite different from the originals. The results illustrate that the robot system mass can be distributed properly among links by optimizing the cross-sectional parameters. Therefore the dynamic performance of flexible manipulators can be improved. In the design of flexible manipulators, the cross-sectional parameters are important variables, and the load capacity of robot can be improved through the optimization of cross-sectional parameters.

Tab. 3 is the result of configurational parameters optimization, i. e. optimization C. It is shown that the mass of the redundant flexible manipulator has been decreased from 375. 76 g to 325. 11 g, the ratio of load to mass has been enlarged by 16% from 0. 133 to 0. 154. We can also find from Fig. 3～Fig. 7 that the joint motion of the manipulator has been changed greatly due to variation in link length, $l_i(i=1,2,3,4)$. Therefore, the kinematic and dynamic performance of the manipulator has been improved. It can be seen in Fig. 3 that the end point deformation error of the flexible manipulator decreases when the manipulator becomes lighter. Therefore the cross-sectional parameters can be further optimized to decrease the mass of manipulator.

In the strategy of comprehensive optimization, i. e. optimization D, the cross-sectional parameters and configurational parameters are optimized simultaneously. The distribution of the manipulator system mass becomes more reasonable, the mass of the manipulator further decreases to 134. 24 g, a decrease of 40% and 58. 7% as compared with optimization B and optimization C, respectively. The ratio of load to mass of robot reaches 0. 357, more than 1/3. This result is nearly impossible for rigid robots or non-redundant flexible manipulators. It is shown that the load carrying capacity of the flexible manipulator has been improved greatly through optimization D.

From the results of optimization, we can also find that the link dimension becomes smaller gradually from the base to the end of manipulator, which helps the robot behave with the maximum stiffness and minimum mass. This reasonable result can be useful to guide the design of flexible manipulators.

5 Conclusions

The design parameter optimization of redundant flexible manipulators has been investigated through theoretical and numerical analysis. It has been illustrated that both the cross-sectional parameters and configurational parameters are important design variables for redundant flexible manipulators. It is necessary for the improvement of dynamic performance of flexible manipulators to optimize the configurational parameters and cross-sectional parameters simultaneously. The design strategies proposed in this study have been shown to be greatly effective in improving the load capacity of flexible redundant manipulators.

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